

1. Fig. 8 shows the curve $y = f(x)$, where $f(x) = (1 - x)e^{2x}$, with its turning point P.

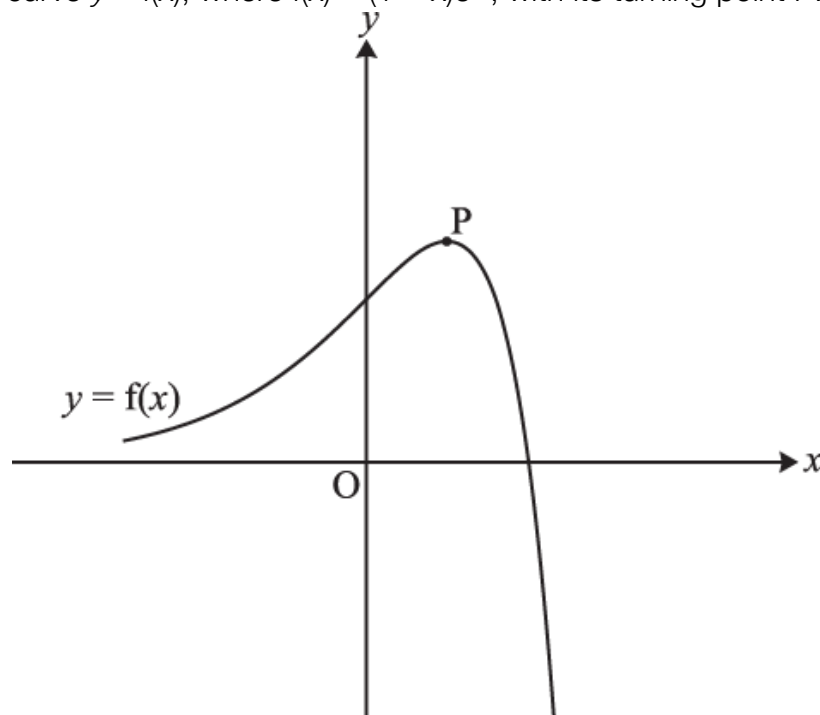


Fig. 8

- i. Write down the coordinates of the intercepts of $y = f(x)$ with the x - and y -axes. [2]
- ii. Find the exact coordinates of the turning point P. [6]
- iii. Show that the exact area of the region enclosed by the curve and the x - and y -axes is $\frac{1}{4}(e^2 - 3)$. [5]

The function $g(x)$ is defined by $g(x) = 3f\left(\frac{1}{2}x\right)$.

- iv. Express $g(x)$ in terms of x .
Sketch the curve $y = g(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x - and y -axes and of its turning point. [4]
- v. Write down the exact area of the region enclosed by the curve $y = g(x)$ and the x - and y -axes. [1]

2. Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where m is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

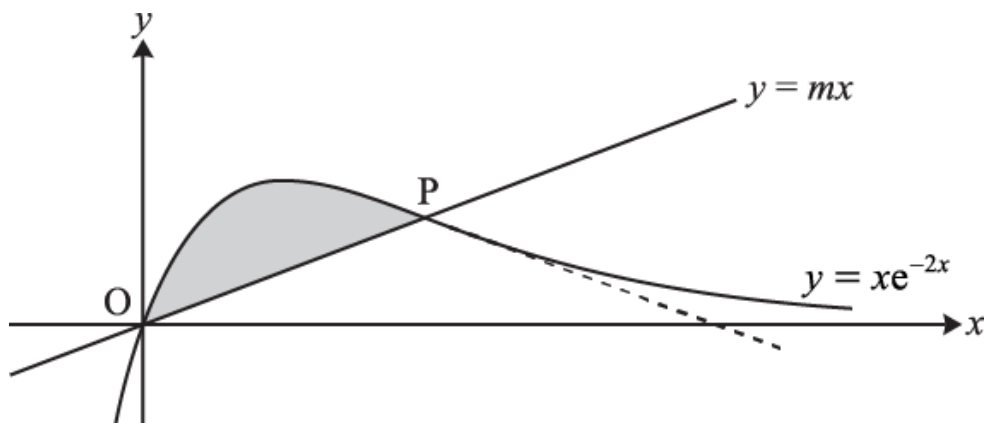


Fig. 9

- i. Show that the x -coordinate of P is $-\frac{1}{2} \ln m$.

[3]

- ii. Find, in terms of m , the gradient of the tangent to the curve at P.

[4]

You are given that OP and this tangent are equally inclined to the x -axis.

- iii. Show that $m = e^{-2}$, and find the exact coordinates of P.

[4]

- iv. Find the exact area of the shaded region between the line OP and the curve.

[7]

3. Find the exact value of $\int_1^2 x^3 \ln x \, dx$.

[5]

4. Find $\int_1^4 x^{-\frac{1}{2}} \ln x \, dx$, giving your answer in an exact form.

[5]

5. Find $\int x^2 e^{2x} dx$. [7]
6. Find $\int 4x^2 \sin 2x dx$. [6]
7. Using the substitution $x = e^u$, find $\int (\ln x)^2 dx$. [6]
8. Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

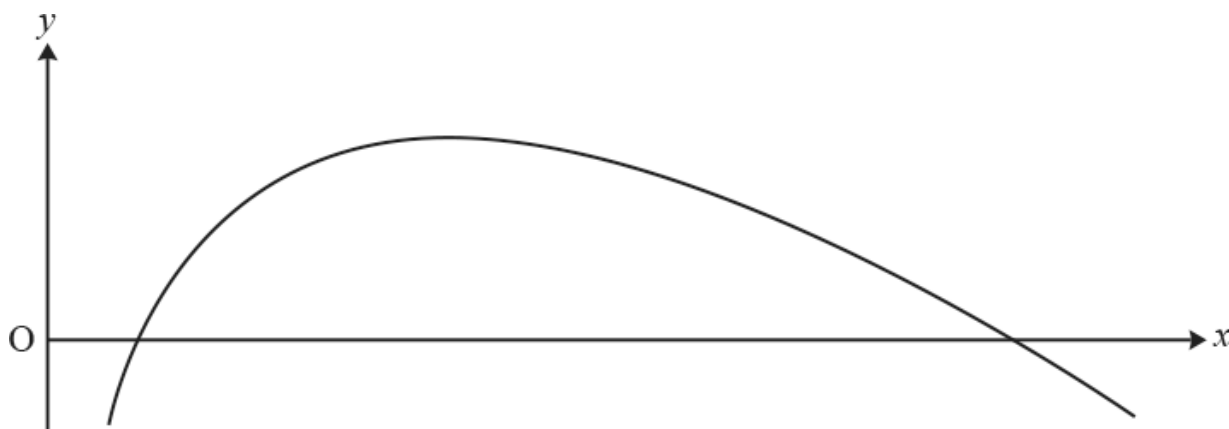


Fig. 10

Find, in terms of k , the area of the finite region between the curve and the x -axis. [8]

9. **In this question you must show detailed reasoning.**

Fig. 15 shows the graph of $y = 5xe^{-2x}$. Find the exact value of the area of the shaded region between the curve, the x -axis and the line parallel to the y -axis through the maximum point on the curve. [9]

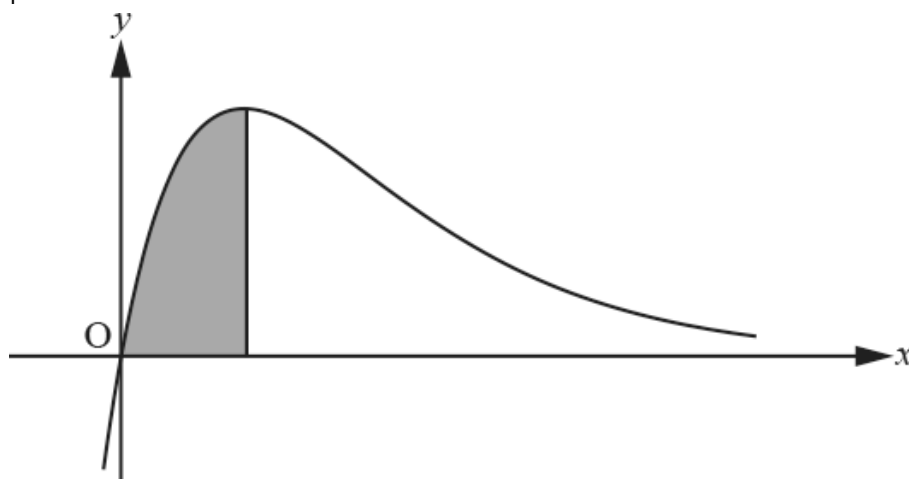


Fig. 15

10. In this question you must show detailed reasoning.

The curve $y = \ln x$ passes through the point (a, b) , where $a > 1$.

The area A is bounded by the x -axis, the line $x = a$ and the curve $y = \ln x$.

The area B is bounded by the x -axis, the y -axis, the line $y = b$ and the curve $y = \ln x$.

The area A is equal in magnitude to the area B .

- (a) Show that a satisfies the equation $pa \ln a + qa + r = 0$, where p , q and r are constants to be determined.

[7]

The value of a is found using the Newton-Raphson method on a spreadsheet. The output is shown in Fig. 15.

| r | x_r |
|-----|----------|
| 0 | 4 |
| 1 | 5.177399 |
| 2 | 4.931531 |
| 3 | 4.921571 |
| 4 | 4.921554 |
| | |

Fig. 15

Heidi states that the value of a is 4.921554 correct to 6 decimal places.

- (b) Determine whether she is correct.

[2]

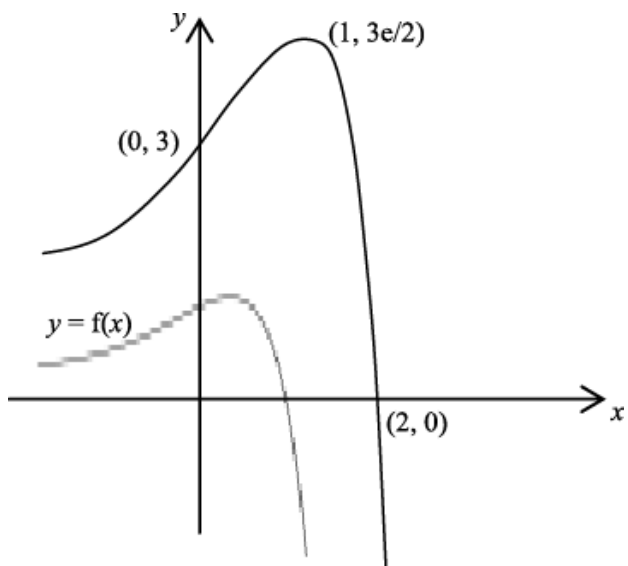
END OF QUESTION paper

Mark scheme

| Question | | Answer/Indicative content | Marks | Part marks and guidance |
|----------|----|------------------------------------|-------|---|
| 1 | i | (1, 0) and (0, 1) | B1B1 | $x = 0, y = 1; y = 0, x = 1$ <u>Examiner's Comments</u> The points of intersection were a write-down for many candidates. Weaker attempts failed to solve $(1 - x)e^{2x} = 0$ convincingly. |
| | ii | $f'(x) = 2(1 - x)e^{2x} - e^{2x}$ | B1 | $d/dx(e^{2x}) = 2e^{2x}$ |
| | ii | | M1 | product rule consistent with their derivatives |
| | ii | $= e^{2x}(1 - 2x)$ | A1 | correct expression, so $(1 - x)e^{2x} - e^{2x}$ is BOM1A0 |
| | ii | $f'(x) = 0$ when $x = \frac{1}{2}$ | M1dep | setting their derivative to 0 dep 1 st M1 |
| | ii | | A1cao | $x = \frac{1}{2}$ allow $\frac{1}{2} e^1$ isw |
| | ii | $y = \frac{1}{2} e$ | B1 | <u>Examiner's Comments</u> This proved to be an accessible 6 marks for candidates. The derivative of e^{2x} and the product rule were generally correct, and deriving $x = \frac{1}{2}$ and $y = e^{1/2}$ was straightforward, though many did not simplify the derivative to $e^{2x} - 2xe^{2x}$ immediately. Some candidates approximated for $e^{1/2}$ and lost a mark. |

| | | | | Integration by Parts |
|--|-----|---|-------|--|
| | iii | $A = \int_0^1 (1-x)e^{2x} dx$ | B1 | correct integral and limits; condone no dx (limits may be seen later) |
| | iii | $u = (1-x), u' = -1, v' = e^{2x}, v = \frac{1}{2} e^{2x}$ | M1 | u, u', v, v' all correct; or if split up $u = x, u' = 1, v = e^{2x}, v' = \frac{1}{2} e^{2x}$ |
| | iii | $\Rightarrow A = \left[\frac{1}{2}(1-x)e^{2x} \right]_0^1 - \int_0^1 \frac{1}{2} e^{2x} \cdot (-1) dx$ | A1 | condone incorrect limits; or, from above, $\dots \left[\frac{1}{2} x e^{2x} \right]_0^1 - \int_0^1 \frac{1}{2} e^{2x} dx$ |
| | iii | $= \left[\frac{1}{2}(1-x)e^{2x} + \frac{1}{4} e^{2x} \right]_0^1$ | A1 | o.e. if integral split up; condone incorrect limits |
| | iii | $= \frac{1}{4} e^2 - \frac{1}{2} - \frac{1}{4}$ | | |
| | iii | $= \frac{1}{4} (e^2 - 3) *$ | A1cao | NB AG <u>Examiner's Comments</u> Most candidates applied integration by parts to either $\int (1-x)e^{2x} dx$ or $\int x e^{2x} dx$, using appropriate u, v, u' and v' . Sign and/or bracket errors sometimes meant they failed to derive the correct result, but many were fully correct. |
| | iv | $g(x) = 3(\frac{1}{2} x) = 3(1 - \frac{1}{2} x) e^x$ | B1 | o.e; mark final answer |

iv



through (2,0) and (0,3) – condone errors in writing coordinates (e.g. (0,2)).

reasonable shape, dep previous B1

B1

TP at (1, 3e/2) or (1, 4.1) (or better).
(Must be evidence that $x = 1, y = 4.1$ is indeed the TP – appearing in a table of values is not enough on its own.)

B1dep

Examiner's Comments

B1

This part proved to be quite demanding. Deriving the formula for $g(x)$ was rarely correctly done. Common errors were an extra factor of 3 and an incorrect exponent. Most graphs showed the correct points of intersection (0, 3) and (2, 0), but the turning point was quite often incorrect or missing, and the shape failed to convince.

v

$6 \times \frac{1}{4} (e^2 - 3) [= 3(e^2 - 3)/2]$

o.e. mark final answer

Examiner's Comments

B1

Those, of the relatively few candidates, who got this correct just wrote down $2 \times 3 \times \frac{1}{4} (e^2 - 3)$. Some tried to integrate $g(x)$, with little success.

Total

18

| | | | | Integration by Parts |
|-----|---|------|--|--|
| | | | here. The main error was to get a derivative of $-2xe^{-2x}$. | |
| iii | $m + m \ln m = -m$ | M1 | their gradient from (ii) = $-m$ | |
| iii | $\Rightarrow \ln m = -2$ | | | |
| iii | $\Rightarrow m = e^{-2}$ | A1 | NB AG | |
| iii | <i>or</i> | | | |
| iii | $y + \frac{1}{2} m \ln m = m(1 + \ln m)(x + \frac{1}{2} \ln m)$ $x = -\ln m$, $y = 0 \Rightarrow \frac{1}{2} m \ln m = m(1 + \ln m)(-\frac{1}{2} \ln m)$ $\Rightarrow 1 + \ln m = -1$, $\ln m = -2$, $m = e^{-2}$ | B2 | for fully correct methods finding xintercept of equation of tangent and equating to $-\ln m$ | |
| iii | At P, $x = 1$ | B1 | | |
| | | | isw approximations | |
| | | | Examiner's Comments | |
| iii | $\Rightarrow y = e^{-2}$ | B1 | The first two marks here were the least successfully answered, because most candidates were not familiar with the fact that lines equally inclined to the x-axis have gradients m and $-m$. Only the best candidates found the result successfully. However, many recovered to find the coordinates of P correctly. | not $e^{-2} \times 1$ |
| iv | Area under curve $= \int_0^1 x e^{-2x} dx$ | | | |
| iv | $u = x$, $u' = 1$, $v' = e^{-2x}$, $v = -\frac{1}{2} e^{-2x}$ | M1 | parts, condone $v = k e^{-2x}$, provided it is used consistently in their parts formula | ignore limits until 3 rd A1 |
| iv | $= \left[-\frac{1}{2} x e^{-2x} \right]_0^1 + \int_0^1 \frac{1}{2} e^{-2x} dx$ | A1ft | ft their v | |

| | | | | |
|----------|---|--|--|---|
| | <p>iv $= \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^1$</p> <p>iv $= (-\frac{1}{2} e^{-2} - \frac{1}{4} e^{-2}) - (0 - \frac{1}{4} e^0)$ $[= \frac{1}{4} - \frac{3}{4} e^{-2}]$</p> <p>iv Area of triangle = $\frac{1}{2}$ base \times height</p> <p>iv $= \frac{1}{2} \times 1 \times e^{-2}$</p> <p>iv So area enclosed = $\frac{1}{4} - 5e^{-2}/4$</p> | <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1cao</p> | <p>$-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}$ o.e</p> <p>correct expression</p> <p>fit their 1, e^{-2} or $[e^{-2}x^2/2]$</p> <p>o.e. must be exact, two terms only</p> <p>Examiner's Comments</p> <p>This question tested the more able candidates. The integration by parts required careful control of negative signs and accurate work; the area of the triangle (or integral of the line) were quite often discernable from the working, which was often scrambled and incoherent – perhaps because some candidates were rushing to complete the paper!</p> | <p>need not be simplified</p> <p>o.e. using isosceles triangle</p> <p>M1 may be implied from 0.067 ...</p> <p>isw</p> |
| | <p>Total</p> | <p>18</p> | | |
| <p>3</p> | <p>let $u = \ln x$, $dv/dx = x^3$, $du/dx = 1/x$, $v = \frac{1}{4} x^4$</p> <p>$\int_1^2 x^3 \ln x \, dx = \left[\frac{1}{4} x^4 \ln x \right]_1^2 - \int_1^2 \frac{1}{4} x^4 \cdot \frac{1}{x} \, dx$</p> <p>$= \left[\frac{1}{4} x^4 \ln x \right]_1^2 - \int_1^2 \frac{1}{4} x^3 \, dx$</p> | <p>M1</p> <p>A1</p> <p>M1dep</p> | <p>u, v, dv, v all correct</p> <p>$\frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \cdot \frac{1}{x} [dx]$</p> <p>simplifying $x^4/x = x^3$ in second term (soi)</p> | <p>ignore limits</p> <p>dep 1st M1</p> |

| | | | | |
|---|--|--|--|--|
| | | $= \left[\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 \right]_1^2$ $= 4 \ln 2 - 15/16$ | <p>A1cao</p> $\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 \text{ o.e.}$ <p>o.e. must be exact, but can isw</p> <p>Examiner's Comments</p> <p>There was a pleasing response to this question. Integration by parts was well understood by the majority of candidates, many of whom gained full marks. Very occasionally, u and v' were allocated to the wrong parts, and the other most common error was failing to simplify v u' before integrating this.</p> <p>A1cao</p> | <p>Integration by Parts</p> <p>must evaluate $\ln 1 = 0$ and combine $-1 + 1/16$</p> |
| | Total | | 5 | |
| 4 | <p>let $u = \ln x$, $u' = 1/x$, $v' = x^{-1/2}$, $v = kx^{1/2}$</p> $\int x^{-1/2} \ln x [dx] = [2x^{1/2} \ln x] - \int 2x^{1/2} \cdot \frac{1}{x} [dx]$ $= [2x^{1/2} \ln x] - \int 2x^{-1/2} [dx]$ $= [2x^{1/2} \ln x - 4x^{1/2}]_1^4$ $= 4 \ln 4 - 8 - (2 \ln 1 - 4)$ $= 4 \ln 4 - 4$ | <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1cao</p> | <p>soi ($k \neq 0$)</p> <p>$x^{1/2} / x = x^{-1/2}$ or $1/x^{1/2}$ seen</p> <p>$2x^{1/2} \ln x - 4x^{1/2}$</p> <p>oe (eg $\ln 256 - 4$) but must evaluate $\ln 1 = 0$</p> | <p>may be integrated separately</p> <p>mark final answer</p> <p>Examiner's Comments</p> <p>Integration by parts was well understood, with just under half candidates scoring full marks for this question. Very occasionally, candidates</p> |

took $u = x^{-1/2}$ and $v = \ln x$, and were unable to score any marks. With u and v correct, the next hurdle is to simplify the $2x^{1/2} \cdot 1/x$ integrand, and some failed at this stage, and attempted to integrate the product term by term. Having negotiated this successfully, most got full marks, though very occasionally the final answer was spoiled by using $4\ln 4 = \ln 16$.

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|--|--|-------|---|--|--|
| | | | | | |
| | | Total | 5 | | |

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|---|--|--|--|--------|--|
| 5 | | <p>let $u = x^2, u' = 2x, v = e^{2x},$</p> $v = \frac{1}{2}e^{2x}$ $\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int 2x \cdot \frac{1}{2}e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int x e^{2x} dx$ | M1A1(AO1.1a 1.2) | | |
| | | <p>let $u = x, u' = 1, v = e^{2x},$</p> $v = \frac{1}{2}e^{2x}$ $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $= \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} (+c)$ </div> <p>so $\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + c$</p> | A1(AO1.1) M1(AO1.1a) A1(AO1.1) A1(AO1.1) A1(AO2.5) | | |
| | | | [7] | Do not | |

| | |
|----------------------|--|
| award if no '+ c' | |
|----------------------|--|

| | | | |
|-------|---|--|--|
| Total | 7 | | |
|-------|---|--|--|

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|---|--|--|--|
| 6 | $-4x^2 \times \frac{\cos 2x}{2} - \int (-8x) \times \frac{\cos 2x}{2} dx$ $4x \times \frac{\sin 2x}{2} - \int 2 \sin 2x dx$ $-2x^2 \cos 2x + 2x \sin 2x - \int 2 \sin 2x dx$ $-2x^2 \cos 2x + 2x \sin 2x - \left(\frac{-2 \cos 2x}{2} \right) + c$ $-2x^2 \cos 2x + 2x \sin 2x + \cos 2x + c$ | <p>M1(AO3.1a)</p> <p>A1(AO1.1b)</p> <p>M1(AO2.1)</p> <p>A1(AO1.1b)</p> <p>M1(AO2.1)</p> <p>A1(AO1.1b)</p> <p>[6]</p> | <p>Integration by parts, allow sign errors only for M1</p> <p>all correct</p> <p>Integration by parts, allow sign errors only for M1</p> <p>convincing attempt, allow sign error and/or omission of + c at this</p> |
|---|--|--|--|

| | | | | | | |
|---|--|--|--|--|--|----------------------|
| | | | | stage | | Integration by Parts |
| | | | | all correct AG | | |
| | | Total | 6 | | | |
| 7 | | <p>Use $x = e^u$ and $\frac{dx}{du} = e^u$</p> $\int (\ln x)^2 dx = \int (\ln e^u)^2 e^u du = \int u^2 e^u du$ <p>Use integration by parts</p> $= u^2 e^u - \int 2ue^u du$ $= u^2 e^u - 2(ue^u - \int u^2 e^u du)$ $= u^2 e^u - 2ue^u + 2e^u + c$ $= x(\ln x)^2 - 2x \ln x + 2x + c$ | <p>M1(AO1.1b)</p> <p>A1(AO1.1b)</p> <p>M1(AO3.1a)</p> <p>A1(AO1.1b)</p> <p>M1(AO1.1a)</p> <p>A1(AO1.1b)</p> <p>[6]</p> | <p>Using substitution including dx</p> <p>Simplifying correctly</p> <p>First use of integration by parts</p> <p>First stage all correct</p> <p>Second use of integration by parts</p> <p>Must be in terms of x for final mark</p> | | |

| | Total | 6 | | | | | |
|--------------|---|---|--|--------------|-------------------------|--|--|
| 8 | <p>Curve crosses the x-axis when $y = 0$</p> <p>$y = (k - x)\ln x = 0$</p> <p>Either $k - x = 0$ or $\ln x = 0$</p> <p>$x = k$ or 1</p> <p>EITHER</p> <p>Area = $\int_1^k (k - x)\ln x \, dx$</p> <p>Let $u = \ln x, \frac{dv}{dx} = k - x, \frac{du}{dx} = \frac{1}{x}, v = kx - \frac{1}{2}x^2$</p> <p>Area = $\left[\left(kx - \frac{1}{2}x^2 \right) \ln x \right]_1^k - \int_1^k \frac{1}{x} \left(kx - \frac{1}{2}x^2 \right) dx$</p> <p>$\left[\left(kx - \frac{1}{2}x^2 \right) \ln x \right]_1^k - \int_1^k \left(k - \frac{1}{2}x \right) dx$</p> <p>$\left[\left(kx - \frac{1}{2}x^2 \right) \ln x - \left(kx - \frac{1}{4}x^2 \right) \right]_1^k$</p> | <p>M1 (AO 3.1a)</p> <p>A1 (AO 1.1b)</p> <p>M1 (AO 2.1)</p> <p>A1 (AO 1.1b)</p> <p>M1 (AO 3.1a)</p> <p>A1 (AO 1.1b)</p> <p>M1dep (AO 1.1a)</p> <p>A1 (AO 1.1b)</p> | <p>Attempt to solve $y = 0$</p> <p>Both roots required</p> <p>Using integration by parts with</p> <table border="1" data-bbox="1256 1110 1480 1198"> <tr> <td>$u = \ln x,$</td> <td>$\frac{dv}{dx} = k - x$</td> </tr> </table> <p>clearly argued</p> <p>Allow without limits</p> | $u = \ln x,$ | $\frac{dv}{dx} = k - x$ | | |
| $u = \ln x,$ | $\frac{dv}{dx} = k - x$ | | | | | | |

$$\left(\left(k^2 - \frac{1}{2}k^2 \right) \ln k - \left(k^2 - \frac{1}{4}k^2 \right) \right) \\ - \left(\left(k - \frac{1}{2} \right) \ln 1 - \left(k - \frac{1}{4} \right) \right) \\ = \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 + k - \frac{1}{4}$$

OR Integral split into two separate integrals

$$\int_1^k k \ln x \, dx$$

Let $u = \ln x, \quad \frac{dv}{dx} = k, \quad \frac{du}{dx} = \frac{1}{x}, \quad v = kx$

$$= [kx \ln x]_1^k - \int_1^k \frac{1}{x} kx \, dx$$

$$[kx \ln x]_1^k - \int_1^k k \, dx = [kx \ln x - kx]_1^k$$

$$(k^2 \ln k - k^2) - (k \ln 1 - k) = k^2 \ln k - k^2 + k$$

And

[8]

Simplifying the integrand

Second part correct

M1

Using limits. Dependent on M mark for integration by parts

Cao

M1

M1dep

Using integration by parts with $u = \ln x,$

| | |
|-------------------------|-----------------|
| $\frac{dv}{dx} = k$ | or $u = \ln x,$ |
| $\frac{dv}{dx} = \pm x$ | clearly |

$$\text{Area} = \int_1^k x \ln x \, dx$$

$$\text{Let } u = \ln x, \quad \frac{dv}{dx} = x, \quad \frac{du}{dx} = \frac{1}{x}, \quad v = \frac{1}{2}x^2$$

$$= \left[\frac{1}{2}x^2 \ln x \right]_1^k - \int_1^k \frac{1}{x} \times \frac{1}{2}x^2 dx$$

$$\left[\frac{1}{2}x^2 \ln x \right]_1^k - \int_1^k \frac{1}{2}x \, dx$$

$$\left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right]_1^k$$

$$\left(\frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right) = \frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 + \frac{1}{4}$$

$$\text{Area} = \left(k^2 \ln k - k^2 + k \right) - \left(\frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 + \frac{1}{4} \right)$$

$$= \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 + k - \frac{1}{4}$$

A1

argued

Simplifying the integrand

A1

Substitution of limits seen in at least one integral.

Dependent on M mark for integration by parts

A1

Both integrals correct at this stage Allow without limits

Both integrals fully correct Allow without limits

Cao

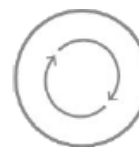
Examiner's Comments

This is an exemplar of a question requiring an extended answer – there are 4 method marks in the scheme. Candidates had to structure their answer. Had a value for k been given, the definite integral would have been possible on many calculators and the question may have become a “detailed reasoning” question.

Successful candidates generally used

| | | |
|---------------------------|-------------------------|----|
| integration by parts with | $\frac{dv}{dx} = k - x$ | It |
|---------------------------|-------------------------|----|

needed much more work to expand the bracket and split the integral in two. Some candidates lost the final mark, as they did not tidy up their answer so had too many terms.



AFL In an unstructured question like this one, do not give up or leave it blank because you do not know how to calculate the limits. Find the indefinite integral to make sure of 4 out of 8 marks. Using

incorrect limits could also have been credited a method mark.

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$$\frac{dy}{dx} = 5x(-2e^{-2x}) + 5e^{-2x}$$

$$e^{-2x}(-10x + 5) = 0$$

$$x = \frac{1}{2}$$

$$\text{Area} = \int_0^{\frac{1}{2}} 5xe^{-2x} dx$$

$$= \left[5x \left(-\frac{1}{2} e^{-2x} \right) \right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \left(-\frac{5}{2} e^{-2x} \right) dx$$

M1 (AO 3.1a)

A1 (AO 1.1)

M1 (AO 1.1a)

A1 (AO 1.1)

M1 (AO 3.1a)

M1 (AO 1.1a)

A1 (AO 1.1)

A1 (AO 1.1)

A1 (AO 1.1)

[9]

For use of product rule

For correct (unsimplified) derivative

For equating derivative to zero and attempting to solve for x

oe

Limits 0 and (their)

$\frac{1}{2}$ must be seen at

some stage for this mark to be awarded

For

9

| | | | | | | | | | | | | | | | |
|---|---|---|------------|--|---|-------------------------------|---------|---|---|---|--|--|---|--|--|
| | | $= -\frac{5}{4}e^{-1} + \left[-\frac{5}{4}e^{-2x}\right]_0^{\frac{1}{2}}$ $= -\frac{5}{2}e^{-1} + \frac{5}{4}$ | | <p>integration by parts with $u = 5x$ and $v = e^{-2x}$ First stage all correct; ignore limits for this mark</p> <p>Integration completed correctly</p> <p>oe, but must be exact simplified form</p> | <p>Integration by Parts</p> <p>Not e.g. 0.3303...</p> | | | | | | | | | | |
| | | Total | 9 | | | | | | | | | | | | |
| 10 | a | <table border="1" style="width: 100%;"> <tr> <td style="width: 50%;">ln $a = b$</td> <td></td> </tr> <tr> <td>Area B:</td> <td>$\int_0^b e^y dy = e^b - e^0$</td> </tr> </table> <p>$= a - 1$</p> <table border="1" style="width: 100%;"> <tr> <td style="width: 50%;">Area A:</td> <td>$\int_1^a \ln x dx = [x \ln x - x]_1^a$</td> </tr> </table> | ln $a = b$ | | Area B: | $\int_0^b e^y dy = e^b - e^0$ | Area A: | $\int_1^a \ln x dx = [x \ln x - x]_1^a$ | <p>B1 (AO 1.1)</p> <p>M1(AO 3.1a)</p> <p>A1(AO 1.1)</p> <p>M1(AO 2.1)</p> <p>A1(AO 1.1)</p> <p>M1(AO 3.1a)</p> <p>A1(AO 1.1)</p> <p>[7]</p> | <table border="1" style="width: 100%;"> <tr> <td style="width: 50%;"></td> <td style="width: 50%;"></td> </tr> <tr> <td>or $a \times b -$ their $a \ln a - a + 1$</td> <td></td> </tr> </table> | | | or $a \times b -$ their $a \ln a - a + 1$ | | |
| ln $a = b$ | | | | | | | | | | | | | | | |
| Area B: | $\int_0^b e^y dy = e^b - e^0$ | | | | | | | | | | | | | | |
| Area A: | $\int_1^a \ln x dx = [x \ln x - x]_1^a$ | | | | | | | | | | | | | | |
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| or $a \times b -$ their $a \ln a - a + 1$ | | | | | | | | | | | | | | | |

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| | | $= a \ln a - a + 1$ Their $a - 1 =$ their $a \ln a - a + 1$ $a \ln a - 2a + 2 = 0$ | | | Integration by Parts | | | | |
| | b | Evaluation of their $f(4.921554 - \delta)$ and their $f(4.921554 + \delta)$ eg -0.000000079882 and 0.000000513742 seen to 2 or more sf plus correct conclusion: sign change, so Heidi is correct | M1 (AO 2.1) A1(AO 2.2a) [2] | <table border="1"> <tr> <td>$\delta \leq 0.000\ 000$</td> <td></td> </tr> <tr> <td>5</td> <td></td> </tr> </table> | $\delta \leq 0.000\ 000$ | | 5 | | |
| $\delta \leq 0.000\ 000$ | | | | | | | | | |
| 5 | | | | | | | | | |
| | | Total | 9 | | | | | | |